

**Mathematics Specialist Units 1,2  
Test 6 2017**

**Section 1 Calculator Free  
Complex Numbers, Proof**

**STUDENT'S NAME** \_\_\_\_\_

**DATE:** Thursday 7 September

**TIME:** 50 minutes

**MARKS:** 58

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Express the following recurring decimals as a fraction.

(a)  $0.\overline{123}$  [2]

(b)  $6.\overline{807}$  [3]

2. (3 marks)

Prove the product of three consecutive even whole numbers is a multiple of 8.

3. (6 marks)

(a) Prove  $\sqrt{11}$  is irrational. [3]

(b) Prove  $\log_3 7$  is irrational. [3]

4. (6 marks)

Solve

(a)  $2x^2 + 3x + 7 = 0$  [3]

(b)  $z - 2\bar{z} = 4 + 3i$  (Hint: let  $z = a + bi$ ) [3]

5. (23 marks)

Given  $w = 5 - 4i$  and  $z = -2 + 3i$

(a) Determine

(i)  $z^2$  [2]

(ii)  $w\bar{z}$  [2]

(iii)  $\frac{w}{z}$  [3]

(iv)  $\text{Im}(w + iz)$  [2]

(b) Determine whether  $\bar{w} \times \bar{z} = (\overline{wz})$  [4]

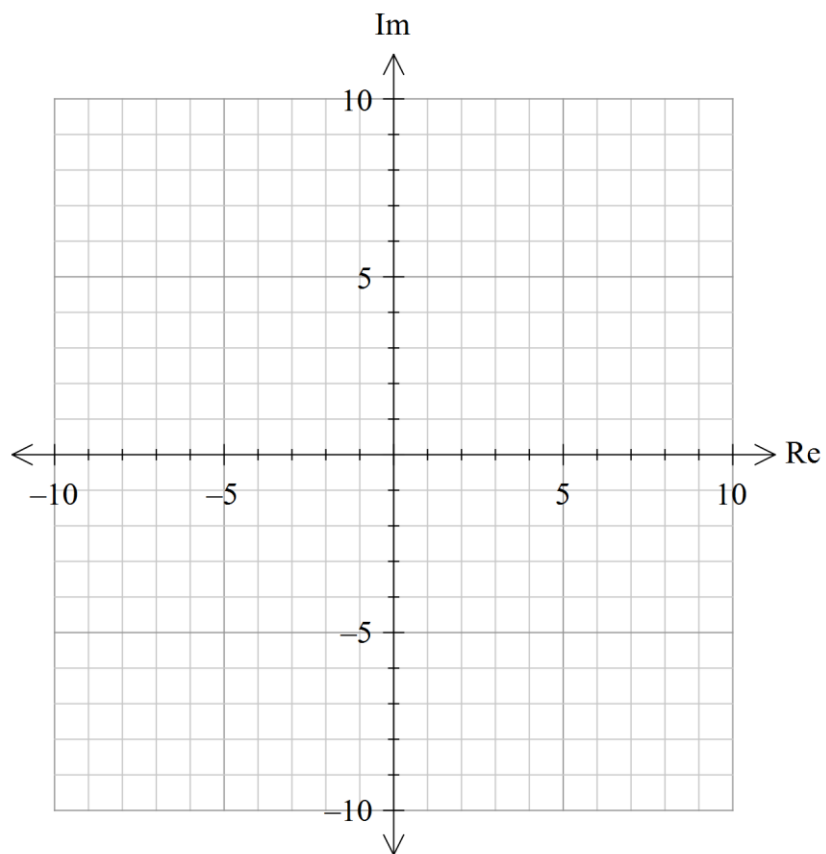
(c) Determine  $a$  and  $b$  if  $a + bi = i(w^{-1})$  [4]

(d) Locate each of the following on the Argand Plane shown.

(i)  $-2i$  [1]

(ii)  $iw$  [2]

(iii)  $\frac{z}{i}$  [3]



6. (12 marks)

Use proof by induction for each of the following.

(a) Prove  $3^{2n} - 1$  is divisible by 8 for integer  $n \geq 1$ . [6]

(b) Prove  $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}^n = \begin{bmatrix} \cos nx & -\sin nx \\ \sin nx & \cos nx \end{bmatrix}$  for integer  $n \geq 1$  [6]

7. (3 marks)

Prove every prime number greater than 4 is either one more or one less than a multiple of 6.



